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ABSTRACT

The use of multiple representations with or without technology is one of the major topics in mathematics education that has gained importance in recent decades. The use of multiple representations is advocated by many mathematics educators and supported by the National Council of Teachers of Mathematics (NCTM) Standards. In this paper, multiple representations are defined as external mathematical embodiments of ideas and concepts to provide the same information in more than one form. It is suggested that multiple representations provide an environment for students to abstract and understand major mathematical concepts. Thus, it is necessary to understand how students see and use these representations. The questions addressed in this study included: (1) what are students' attitudes towards multiple representations?; (2) what affects students' choice of representation to solve a mathematics problem?; and (3) how does the computer setting affect students' choice of representations? After observing a remedial college freshman mathematics class, interviewing the instructor, and having one computer lab hour with the class, a Likert scale questionnaire based on the data collected in the interview and observations, was administered in order to obtain the students' attitudes toward mathematics and multiple representations, their strategies and preferences related to the use of representations, and influences of technology on this issue. Findings indicate that previous knowledge and experience and personal preferences were the main themes in choosing a mathematical representation. (Contains 20 references.) (Author/ASK)

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STUDENTS' USE OF REPRESENTATIONS IN MATHEMATICS EDUCATION*

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Abstract

The use of multiple representations with or without technology is one of the major topics in mathematics education that has been gained importance in recent decades. The use of multiple representations is advocated by many mathematics educators and also supported by the NCTM Standards. Here, multiple representations are defined as external mathematical embodiments of ideas and concepts to provide the same information in more than one form. It is suggested that multiple representations provide an environment for students to abstract and understand major mathematical concepts. Thus, it is necessary to understand how students see and use these representations. The questions of this study were: what are students' attitudes towards multiple representations; what affects students' choice of representation to solve a mathematics problem; and how does the computer setting affect students' choice of representations? After observing a remedial college freshman mathematics class, interviewing the instructor, and having one computer lab hour with this class, a Likert scale questionnaire based on the data collected in the interview and observations, was administered in order to obtain the students' attitudes toward mathematics and multiple representations, their strategies and preferences related to the use of representations, and influences of technology on this issue. Previous knowledge and experience and personal preferences were the main themes for choosing a mathematical representation.

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STUDENTS' USE OF REPRESENTATIONS IN MATHEMATICS EDUCATION

The use of multiple representations is advocated by many mathematics educators and also supported by the NCTM Standards (NCTM, 1989). Many articles call for studies to improve instructional strategies in this area.

> Mathematics instructional programs should emphasize mathematical representations to foster understanding of mathematics so that all students-
>
> - create and use representations to organize, record, and communicate mathematical ideas;
> - develop a repertoire of mathematical representations that can be used purposefully, flexibly, and appropriately;
> - use representations to model and interpret physical, social and mathematical phenomena. (NCTM, 1998, p.91)

There are questions as to how reasoning, problem solving, and learning processes are altered by multiple representations compared to single representation and what are expected outcomes that justify a wide variety of representations. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts (McArthur et al., 1988; Yerushalmy, 1991).

Constructivist theory suggests that we need to understand students' thinking processes in order to facilitate their learning in more empowering ways (Steffe, 1991). The aim of this pre-pilot study was to understand students' thinking and their preferences while choosing a representation type for solving a mathematical problem and to help educators see some effects of student thinking and their rationalizing process when students are dealing with mathematical representations.

Briefly, the questions of this study were:

a) What are students' attitudes towards multiple representations?

b) What affects their choice of representation to solve any problem?

c) How does a computer setting affect their choice of representations?

Conceptual Framework

> *Because each representation emphasizes and suppresses various aspects of a concept, we believe that students gain a more thorough understanding of a function if it is explored using numerical, graphical, and analytical methods (Piez and Voxman, 1997, p.164).*

There are different definitions for "representation" in mathematics education. Most researchers differentiate between external and internal representations where external representations are embodiments of ideas or concepts such as charts, tables, graphs, diagrams, etc., and internal representations are cognitive models that a person has (Janvier et al., 1993). In this study, multiple representations are external mathematical embodiments of ideas and concepts to provide the same information in more than one form.

Dufour-Janvier et al. (1987) identify the role of representations in mathematics education by several characteristics as follows:

- Representations are an inherent part of mathematics.
- Representations are multiple concretizations of a concept.
- Representations are used locally to mitigate certain difficulties.
- The representations are intended to make mathematics more attractive and interesting.(pp. 110-111)

Although there are a number of theories that address multiple representations, it was with Dienes' "multiple embodiment principle" that this issue gained importance. The Multi-embodiment Principle or Perceptual Variability Principle suggests that the mathematical concepts should be presented in as many different forms as possible in

order for students to obtain the mathematical essence of an abstraction. Dienes also mentioned that conceptual learning is maximized when children are exposed to a mathematical concept through a variety of physical contexts or embodiments (Dienes, 1960).

Constructivist theory suggests that students must construct their knowledge by themselves *actively* (Goldin, 1990; Noddings, 1990; von Glasersfeld, 1990, 1993, 1996). As von Glasersfeld (1996) mentions, radical constructivism suggests that we construct our knowledge in our experiential world constituted by our own ways and means of perceiving and no knowledge can claim *uniqueness*. Therefore, we should not expect that everyone will perceive the same concept from one representation.

Studies from Donnelly (1995), Dufour-Janvier, Bednarz, and Belanger (1987), Eisenberg and Dreyfus (1991), Poppe (1993), Porzio (1994), and Vinner (1989) have been synthesized to gain a have deeper understanding about students' ways of using multiple representations in mathematics classrooms. Two main categories have been identified: internal and external effects. Table 1 is an integration of the findings from the studies mentioned above.

In this study, I wanted to focus on students' *own* thoughts, beliefs, and attitudes towards representations in computer and noncomputer settings. If we learn about these issues, I believe it will help us understand the learning process with representations.

Table 1. Reasons for Students' Preferences for Representations

Internal Effects	Personal Preferences Previous Experience Previous Knowledge Beliefs about mathematics Rote Learning
External Effects	Presentation of Problem Problem itself Sequential Mathematics Curriculum Dominance of algebraic representation in teaching Technology and graphing utilities

Methodology

After observing a remedial college freshman mathematics class, interviewing the instructor, and having one computer lab hour with this class, a survey was developed based upon data collected and relevant literature.

Site

The site was a remedial mathematics class at a large midwestern university. There were 16 students, 9 female and 7 male. This class was taught by a doctoral student in mathematics education. The ultimate goal for this class was for students to change their beliefs and attitudes about mathematics and make them believe that they can do mathematics. When I asked the instructor more information about this class she stated:

> It is the remedial math 075 and last quarter was 050. It is a special section where we use more of the research done in math education on learning and teaching to work with these students. Things that we do a little bit different [are] we have a smaller class size. Our maximum is 24. We do more hands-on activities. We work more with the constructivist ideas of starting with what they know and building upon that knowledge. That is I think the biggest. Our assessment is different of course because like the research says you can't have instruction one way and the assessment the other way. They have to be in alignment. Our pedagogy, our curriculum and assessment are a little bit different. As far as topics [go], we cover basically the same thing that is in the regular 050 and 075. (personal Interview, 1/30/98)

Data Collection Methods

An interview was conducted with the instructor of the classroom about her class in general, the use of technology in mathematics education, and multiple representations. It was a semi-structured interview. It was audio recorded and transcribed.

One observation was conducted in an ordinary class setting where students worked on an activity in groups. The activity consisted of four parts. The first part stated the main question and there was no suggestion related to representations. The aim of this part of the activity was to see which representation students would choose to solve the problem. The second part of this activity involved looking at the same problem graphically and asking questions related to the graphical representation. The third part of the problem did the same for a tabular representation. The final part of the problem emphasized the algebraic form of representation. This activity was chosen purposefully by the instructor in order to help the researcher.

A class session using computer software was also observed. First the researcher introduced the computer program to the students and then the instructor gave a mathematical problem on which students could work using the software. It was hard for students to understand the software and use it effectively in just one class period, but it gave them an idea how they can use computer software to solve mathematics problems.

Finally, a Likert scale questionnaire based on the data collected in the interview and observations, was administered in order to obtain the students' attitudes toward mathematics and multiple representations, their strategies and preferences related to the use of representations, and influences of technology on this issue.

Data Analysis

The first five questions in the questionnaire were related to attitudes towards mathematics. Since they were remedial mathematics students, it was expected that they would have negative attitudes towards mathematics. They agreed that mathematics made them feel uneasy and confused. They do not think that mathematics is enjoyable and stimulating to them. They also agreed that mathematics has been their worst subject. Although the instructor tried to improve these attitudes, it was not easy for them to change overnight. The following quotation from the interview with the instructor shows her efforts to improve this situation:

> One of the things that I changed in this class...[was] I really try to make them think about their thinking about math...At the beginning they always say. "This is how I do it; I just have to memorize", and I have to confront them with [this approach] is not working for you. You would not be in remedial math class if that worked. You've got to change your way of thinking about what is good for you in math. What you have been doing is not good for you and [it] bothers them. Some of them have changed their way of thinking. The ones that have

changed...say, "You know she is right; I need to think about math in a different way." (personal Interview, 1/30/98)

She also mentions this goal in her syllabus for this class:

> My goal for this course, besides covering the necessary topics is to challenge your thinking about mathematics.

The questionnaire also included questions on attitudes towards multiple representations on the questionnaire. Although students agreed that mathematics problems can be solved in various ways by using different representations, most of them (10 out of 14) found it easier to focus on one representation, than to deal with many representations (See Figure 1). At this point, I think that they may not be comfortable with many representations that are presented at the same time.

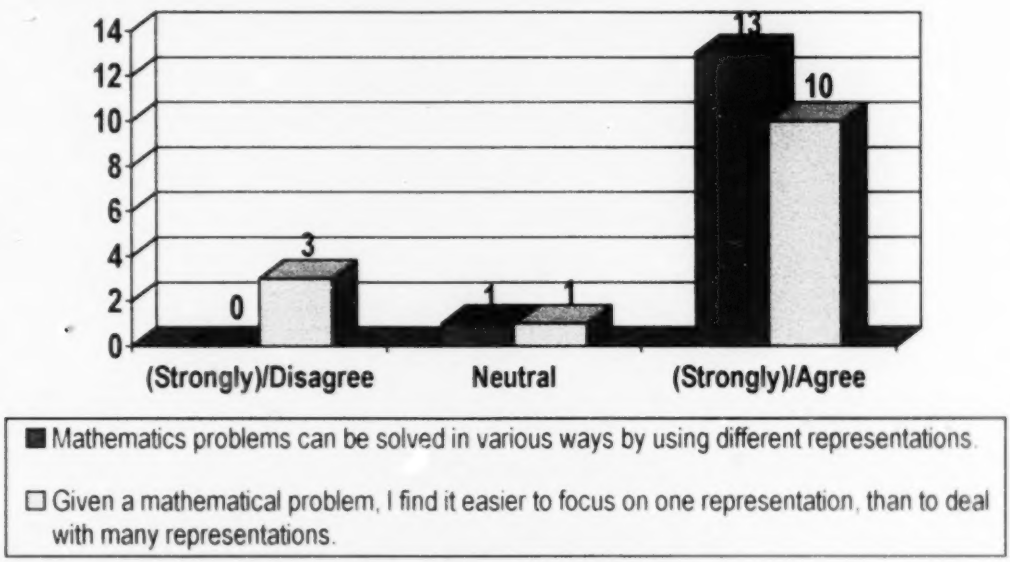

Figure 1. Students attitudes toward multiple representations.

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Questions asking about students' choice of representations while they were solving mathematics problems showed that most of them (10 out of 14) believed that equations and graphs made mathematics problems clearer. Figure 2 shows students' preferences for (a) initial representation when they solve a mathematics problem, (b) the representation they liked while using computer software, and (c) the representation they were most comfortable with.

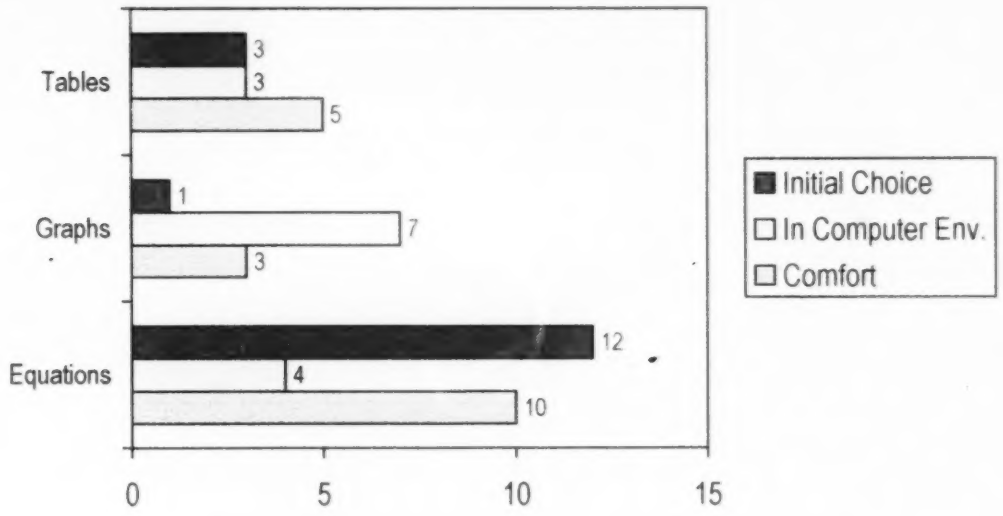

Figure 2. Students' preferences for representations.

Ten students were most comfortable with equations, whereas three of them were most comfortable with graphs and five of them were most comfortable with tables. As I observed in the class, all of them used equations to solve a problem. Moreover, when I asked the instructor in the interview why they mostly use equations, she said, "I think they are still uncomfortable with graphing" (Personal Interview, 1/30/98). She was right; only three of them answered that they are comfortable with graphs.

The questionnaire also contained questions about how computer software affected their choice of representation. Only four of them liked using equations in this

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program while seven of them liked using graphs and seven of them thought that software made graphs easier (see Figure 2). Technology made sketching the graph easier, since by pushing one button they could easily see the graph and visualize the situation. Perhaps they are not comfortable with sketching graphs, but when they have quick access to graphs they can easily use graphs in solving problems. As the instructor mentioned in the interview, "because in graphing you know if their lines are not straight and it gets off on a grid". It is difficult and time consuming to graph by hand; computers make the job much easier.

It was not expected the students would favor the computer software, since it was the first time they used this particular software and some of them were not comfortable with the computer itself. However, I believe that if they had the opportunity to use technology more in their classroom, they would be more comfortable with it. They could use it more effectively and the technology could affect their mathematical thinking and their choice of representation.

Twelve out of the 14 students mentioned that they would start solving mathematics problems using equations, whereas one of them would use with graphs and three of them, tables (see Figure 2). Equations are the dominant representation type for this class. Some students indicated that they are comfortable with equations and they can manage the steps to solve them. Some see the equations as the first step toward sketching the graph.

Previous Knowledge and Experience with the Representation

Looking at the big picture, the major reason for choosing any representation was students' previous knowledge and experience with that representation, as indicated in the following responses to open ended questions on the survey:

> [Graphs] allow me to relate to something I can understand better and it seems to help me better
> [Graph] shows the equations in a way that helps one understand easier
> It all depends on how familiar I am with each process
> Yes, I think if I can understand [in] a certain representation then that is what I will use because if I can understand how to do something a certain way that is easier for me, then that is how I will do it.

Of course, being uncomfortable with a representation was a reason for not using it:

> [Graphs are not useful], because sometimes I get lost.
> [Tables] confuse me.

Although some students accepted that they were not comfortable with certain representation, they believe in the usefulness of particular representations.

> Yes, [equations] are [helpful], but not to me because they usually confuse me.
> At times [equations] can be confusing, but they are useful.

Rationales for Specific Representations

Students had special reasons for using each representation. For example, being able to reach the only true exact answer through the use of equations made them comfortable and confident.

> [Equations are useful] To find solutions to unknown questions, knowing there is only one solution. (open-ended survey questions)
> You have an easy to find, definite, tangible answer [with equations]. (open-ended survey questions)

However, there was a student who was comfortable with having many possible solutions and he mentioned that "Because [tables] show you *many possible solutions* [italics added] to one problem that helps you graph the problem" (open-ended survey questions).

According to my observations, interview and survey, equations were the dominant representation.

> Their first choice is *again* [italics added] algebraic form. (observation notes)
> My students that I have had for two quarters went to algebra. Most of them. (personal Interview, 1/30/98)

Equations are always emphasized more than other representational forms in the mathematics curriculum. However, if we make our classrooms richer by providing other types of representations to the students, then they can choose the one that is accessible for them. This instructor was trying to present all forms of representations to students. However, students have years of experience in which equations were emphasized. It is not easy to change this habit in one quarter.

Another common reason for choosing equations is that when you are using equations you can follow ready-made steps to get the solution. This makes the students confident.

> [Equations] Develop *logical* [italics added] thinking skills.
> [Equations] Because it is step process helps me understand why and what I am doing. (open-ended survey questions)

However, if a student who does not want to follow the steps, has other opportunities to use tables or graphs to solve a problem. One student mentioned that

> I think nonlinearly so a graph and tables shows me something I can relate to much better. (open-ended survey questions)

On the other hand, the common reason for using graphs was the visual benefits of graphs. At this point, we can see how different learning styles or multiple intelligences affect our choices in every part of our life, especially in education. Since

graphs make it possible for students to see how equations behave, students explain their rationale for graphs as follows:

> A symbolic representation (or "visual aid") makes some problems easier to grasp. (open-ended survey questions)
> [Graphs] allow you to take an equation and for people like me who are very visual learners, be able to see it. (open-ended survey questions)
> You know when I was asking which is the better buy. They refereed to a graph because they saw it better. (personal Interview, 1/30/98)

Since tables put information in a more organized and categorized way, some of the students mention this issue when they rationalize tables as their choice of representation:

> Having the information categorized and laid out logically helps me assess the problem. (open-ended survey questions)
> All data can be recorded and draw out to make it clearer. (open-ended survey questions)

Another theme was the evidence that they could see the relationships among the representations. They rationalized some representations as helpful in getting other representations.

> [Equations & tables] It helps me to then make the graph and be able to interpret the graph. (open-ended question-Survey)
> [Tables] Help me calculate the graph. (open-ended survey questions)
> [Equation] shows you how the line is going to look. (open-ended survey questions)

Conclusion

There are multiple reasons affecting students when using and selecting mathematical representations. If we, as educators, provide an environment with multiple representations instead of favoring a particular representation, we will enable

students to experience different representations and choose the representation that is most meaningful for them.

At this point technology can help us by making the graphing procedure or the simultaneous display of more than one representation easier. But I am not saying that we can not provide alternate representations without computers or technology. A teacher can use all types of representations in her class without separating them like "now we are learning graphs, now tables and now equations," and can give students the opportunity to decide the representation that they want to use.

How can technology help? Technology can make the graphing procedure easier. Moreover, computers make it possible to see and access three or four representations at the same time. The reasons Fey (1989) lists that computer-based representation environments are unique were :

- Presenting dynamic computer representations that no text or chalkboard diagram can.
- Individual environments for students that are flexible, but at the same time, constrained to give corrective feedback to each individual user whenever appropriate.
- Helping move students from concrete thinking to a more abstract form.
- The versatility of computer graphics.
- The machine accuracy. (pp. 255-256)

However, in this study, technology could not be used effectively given the constraints of the class, but it was there to be employed with many advantages and disadvantages according to students usage.

Another implication from this study is that we, as educators, need to listen to our students and reflect on their answers. They have lots to say to us. For instance "I think nonlinearly...that is why I do not like equations" or "That is what the book and the

instructor showed me." By providing environments in which students can share their thoughts, such as asking questions in assignments about their rationale for their answers, we can learn more about their thinking, their attitudes towards mathematics, and their ways of knowing, which will empower us to improve our educational practices in the future.

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